39. Proposed by SETH PRATT, C. E., Assyria, Michigan.

The pendulum of a clock which gains 6 seconds in 1 hour and 13 minute, makes 6000 vibrations in 1 hour and 9½ minutes. What is the length of the pendulum? And what length should it have to keep true time?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Regarding 1 hour, 13 minutes and 1 hour, 9½ minutes as registered by a clock keeping correct time, $g=32.16$, $\pi=3.1416$, $t=\pi\sqrt{(l/g)}$. Then 1 hour, 9½ minutes = 4170 seconds.

$$\therefore\ t=\tfrac{4}{6}\tfrac{1}{0}\tfrac{7}{0}\tfrac{0}{0}=\tfrac{1}{2}\tfrac{3}{0}\tfrac{9}{0}=\pi\sqrt{\frac{l}{g}}.\qquad \therefore\ l=\frac{(139)^2 g}{(200\,\pi)^2}=1.57393 \text{ ft.}=18.88716 \text{ inches.}$$

1 hour, 13 minutes = 4380 seconds.

$$\frac{4380\times 200}{139}=\text{number of vibrations in 1 hour, 13 seconds.}$$

$$\therefore\ \frac{4380\times 200}{139}=4386 \text{ seconds.}$$

$$\therefore\ t'=\frac{4386\times 139}{4880\times 200}=\frac{731\times 139}{730\times 200}=\pi\sqrt{\frac{l'}{g}}.$$

$$\therefore\ l'=\frac{(731\times 139)^2 g}{(730\times 200\pi)^2}=1.578243 \text{ feet.}$$

$$\therefore\ l'=18.93892 \text{ inches}=\text{length to keep true time.}$$

II. Solution by E. W. MORRELL, Professor of Mathematics in Montpelier Seminary, Montpelier, Vermont.

1 hour and 9½ minutes = 4170 seconds. 4170 seconds ÷ 6000 = .695 seconds, the time of one vibration. From Mechanics $l=t^2g/\pi^2$, whence $l=18.886$ inches, the length of the pendulum. Again, 1 hour and 13 minutes = 4380 seconds. $4380\div.695=876/.139$ = number of vibrations in 1 hour and 13 minutes. As the pendulum gains 6 seconds in that time, $6\div(876/.139)=.834/876=.0095$, the time in seconds gained in one vibration.

∴ .695 seconds + .0095 seconds = .69595 seconds, the time of vibrations of pendulum to keep correct time. Hence by substitutions in the above formula $l=18.9379$ inches, the length of pendulum to keep true time.

[NOTE.—The results sent in with the problem by the Proposer were, 18.89835+ inches, and for true time .036036+ inches longer. Prof. P. S. Berg in his solution obtained for length of pendulum 18.837975 inches, and 22.393 inches as the length to keep true time. EDITOR.]

PROBLEMS.

49. Proposed by J. SCHEFFER, A. M., Hagerstown, Maryland.

Give a general proof that the centre of gravity, or centroid, determines that point from which the sum of the distances to all other points of a given area is the minimum.